

Worked Solutions

Edexcel C4 Paper I

1. (a) $\frac{dy}{d\theta} = \frac{1}{(1 + \cos \theta)}(-\sin \theta), \frac{dx}{d\theta} = 2 \cos 2\theta$

$$\frac{dy}{dx} = \frac{-\sin \theta}{(1 + \cos \theta)2 \cos 2\theta}$$

where $\theta = \frac{\pi}{6}$, gradient =
$$\frac{-\frac{1}{2}}{\left(1 + \frac{\sqrt{3}}{2}\right) \cdot 2 \cdot \frac{1}{2}} = -\frac{1}{2\left(1 + \frac{\sqrt{3}}{2}\right)}$$

$$= -\frac{1}{2 + \sqrt{3}} = -\frac{(2 - \sqrt{3})}{(2 + \sqrt{3})(2 - \sqrt{3})} = \sqrt{3} - 2 \quad (5)$$

(b) gradient = 0 where $\sin \theta = 0$

i.e. where $\theta = 0$

at $\theta = 0, x = 0, y = \ln 2$

gradient is zero at $(0, \ln 2) \quad (3)$

2. $\int A \, dA = \int e^{\frac{1}{10}t} dt$

$$\frac{A^2}{2} = 10e^{\frac{1}{10}t} + c$$

$A = 20, t = 0: \frac{400}{2} = 10 + c, c = 190$

$$\therefore \frac{A^2}{2} = 10e^{\frac{1}{10}t} + 190$$

when $t = 20, \frac{A^2}{2} = 10e^2 + 190$

$$A = 23 \text{ (2 sig. fig.)} \quad (7)$$

3. (a) $\frac{1}{y} \frac{dy}{dx} + 3x^2 - 2 = 0$

$$\frac{dy}{dx} = y(2 - 3x^2)$$

(b) (i) $e^x \frac{dy}{dx} + ye^x + 2y \frac{dy}{dx} = 0$

$$\frac{dy}{dx}(e^x + 2y) = -ye^x$$

$$\frac{dy}{dx} = \frac{-ye^x}{e^x + 2y}$$

at $(0, 3) \frac{dy}{dx} = \frac{-3}{1+6} = -\frac{3}{7}$

(ii) equation of tangent at $(0, 3)$ is $y - 3 = -\frac{3}{7}x$
 $3x + 7y = 21$

4. (a) (i) $\cos 2x = 1 - 2 \sin^2 x$

hence $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

(ii) $\int \sin^2 x = \frac{1}{2} \int (1 - \cos 2x) dx = \frac{1}{2}x - \frac{1}{4} \sin 2x + c$

(b) Integrating by parts,

$$\int_0^{\frac{\pi}{8}} x \frac{d}{dx} \left(-\frac{1}{2} \cos 2x \right) dx = \left[-\frac{x}{2} \cos 2x \right]_0^{\frac{\pi}{8}} + \int_0^{\frac{\pi}{8}} \frac{1}{2} \cos 2x dx$$

$$= \left[-\frac{x}{2} \cos 2x + \frac{1}{4} \sin 2x \right]_0^{\frac{\pi}{8}}$$

$$= -\frac{\pi}{16} \cdot \frac{1}{\sqrt{2}} + \frac{1}{4} \cdot \frac{1}{\sqrt{2}} - 0$$

5. (a) $\vec{OM} = \mathbf{i} + 2\mathbf{j}$, $\vec{ON} = \mathbf{i} + 2\mathbf{k}$

(b) line OM : $\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$

line AB : $\mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

at intersection of OM and AB , $0 + \lambda = 2 + 0 \Rightarrow \lambda = 2$

$0 + 2\lambda = 0 + \mu \Rightarrow \mu = 4$

point of intersection is $\begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix}$.

(c) $\vec{MN} = \begin{pmatrix} 0 \\ -2 \\ 2 \end{pmatrix}$, $|\vec{MN}| = \sqrt{8}$, $\vec{MO} = \begin{pmatrix} -1 \\ -2 \\ 0 \end{pmatrix}$, $|\vec{MO}| = \sqrt{5}$

$\vec{MN} \cdot \vec{MO} = \begin{pmatrix} 0 \\ -2 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -2 \\ 0 \end{pmatrix} = 4$

$\therefore 4 = \sqrt{8}\sqrt{5} \cos \theta$, where $\theta =$ angle required

$\theta = 50.8^\circ$

6. (a) $\int_1^2 \left(2x + \frac{1}{x}\right) dx = [x^2 + \ln x]_1^2 = 4 + \ln 2 - (1 + \ln 1) = 3 + \ln 2$ (3)

(b) (i) $\int x e^x dx = \int x \frac{d}{dx}(e^x) dx = x e^x - \int e^x dx = x e^x - e^x + c$ (3)

(ii) volume $= \pi \int_0^1 x e^x dx = \pi [e^x(x-1)]_0^1 = \pi [e \times 0 - 1(-1)] = \pi$ (4)

(2)

7. (a) $f(x) = \frac{1}{x+3} + \frac{3}{x-1}$ (usu. ...)

(b) $\frac{1}{3+x} = \frac{1}{3\left(1+\frac{x}{3}\right)}$
 $= \frac{1}{3} \left(1+\frac{x}{3}\right)^{-1}$

$f(x) = \frac{1}{3} \left(1+\frac{x}{3}\right)^{-1} - 3(1-x)^{-1}$ [note ...]

$= \frac{1}{3} \left[1 + (-1)\frac{x}{3} + \frac{(-1)(-2)}{2} \frac{x^2}{9} + \dots \right]$

$-3 \left[1 + (-1)(-x) + \frac{(-1)(-2)}{2} (-x)^2 + \dots \right]$

$= \frac{1}{3} - \frac{x}{9} + \frac{x^2}{27} - 3 - 3x - 3x^2$

$= -\frac{8}{3} - \frac{28}{9}x + \frac{80}{27}x^2$

(c) valid for $|x| < 1$ (i.e. $-1 < x < 1$)

(d) $f(x) = (x+3)^{-1} + 3(x-1)^{-1}$

$f'(x) = -(x+3)^{-2} - 3(x-1)^{-2}$

$= -\frac{1}{(x+3)^2} - \frac{3}{(x-1)^2}$

$f'(x) < 0$ as both $(x+3)^2$ and $(x-1)^2$ are always positive

8. (a) $\frac{dy}{dx} = \frac{(1+x^2) \cdot 1 - x \cdot 2x}{(1+x^2)^2}$
 $= \frac{1-x^2}{(1+x^2)^2}$

$\frac{dy}{dx} = 0$ at $1-x^2 = 0$

i.e. $x = 1, -1$

when $x = 1, y = \frac{1}{2}$

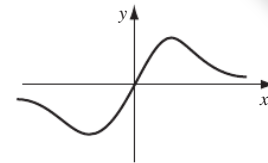
$x = -1, y = -\frac{1}{2}$

(b) $\frac{d^2y}{dx^2} = \frac{(1+x^2)^2(-2x) - (1-x^2)2(1+x^2) \cdot 2x}{(1+x^2)^4}$

when $x = 1, \frac{d^2y}{dx^2} = \frac{-8-0}{2^4}$, which is < 0

$x = -1, \frac{d^2y}{dx^2} = \frac{8-0}{2^4}$, which is > 0

(c)



(5)

(d) $\text{area} = \int_0^2 \frac{x}{1+x^2} dx = \left[\frac{1}{2} \ln(1+x^2) \right]_0^2 = \frac{1}{2} \ln 5$